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AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

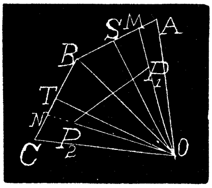
SOLUTIONS OF PROBLEMS.

27. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Penn.

Find the mean area of the *dodecagonal surface* formed by joining in order the points taken at random, one in each *sectoral triangle* of a regular inscribed dodecagon.

Solution by O. W. ANTHONY, Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Let AOB and BOC be two adjacent sectors of the regular dodecagon. Let the dodecagon be determined by its apothem$=a$.



Let $\angle P_1OB=\theta_1$, $\angle P_2OB=\theta_2$, $OP_1=\rho_1$, $OP_2=\rho_2$.

Then area of triangle $P_1OP_2=$

$$\tfrac{1}{2}\rho_1\rho_2\sin(\theta_1+\theta_2).$$

And average area of triangle$=$

$$\varDelta=\tfrac{1}{2}\frac{\int_0^{\frac{1}{6}\pi}\int_0^{\frac{1}{6}\pi}\int_0^{OM}\int_0^{ON}\rho_1\rho_2\sin(\theta_1+\theta_2)d\rho_1d\rho_2d\theta_1d\theta_2}{\int_0^{\frac{1}{6}\pi}\int_0^{\frac{1}{6}\pi}\int_0^{OM}\int_0^{ON}d\rho_1d\rho_2d\theta_1d\theta_2}.$$

$$OM=a\sec(\theta_1-\frac{\pi}{12}).$$

$$ON=a\sec(\theta_2-\frac{\pi}{12}).$$

Then $$\varDelta=\tfrac{1}{2}a^2\frac{\int_0^{\frac{1}{6}\pi}\int_0^{\frac{1}{6}\pi}\sec^2(\theta_1-\frac{\pi}{12})\sec^2(\theta_2-\frac{\pi}{12})\sin(\theta_1+\theta_2)d\theta_1d\theta_2}{\int_0^{\frac{1}{6}\pi}\int_0^{\frac{1}{6}\pi}\sec(\theta_1-\frac{\pi}{12})\sec(\theta_2-\frac{\pi}{12})d\theta_1d\theta_2}.$$

The numerator may be written

$$\int_0^{\frac{1}{6}\pi}\left[\sec^2(\theta_2-\frac{\pi}{12})\int_0^{\frac{1}{6}\pi}\sec^2(\theta_1-\frac{\pi}{12})\sin[(\theta_1-\frac{\pi}{12})+(\theta_2+\frac{\pi}{12})]d\theta_1\right]d\theta_2.$$

The part under the last integral sign may be written, after expansion and some minor reductions,

$$\cos(\theta_2+\frac{\pi}{12})\int_0^{\frac{1}{6}\pi}\sec(\theta_1-\frac{\pi}{12})\tan(\theta_1-\frac{\pi}{12})d\theta_1+\sin(\theta_2+\frac{\pi}{12})\int_0^{\frac{1}{6}\pi}\frac{d\theta_1}{\cos(\theta_1-\frac{\pi}{12})}$$

$$=\cos(\theta_2+\frac{\pi}{12})\int_0^{\frac{1}{6}\pi}\sec(\theta_1-\frac{\pi}{12})+\sin(\theta_2+\frac{\pi}{12})\int_0^{\frac{1}{6}\pi}\log_e\frac{1+\tan\frac{1}{2}(\theta_1-\frac{\pi}{12})}{1-\tan\frac{1}{2}(\theta_1-\frac{\pi}{12})},$$

$$=0+2\sin(\theta_2+\frac{\pi}{12})\log_e\frac{1+\tan\frac{\pi}{24}}{1-\tan\frac{\pi}{24}}.$$

$\therefore$ The numerator may be written:

$$2\log_e\left(\frac{1+\tan\frac{\pi}{24}}{1-\tan\frac{\pi}{24}}\right)\int_0^{\frac{1}{6}\pi}\sec^2(\theta_2-\frac{\pi}{12})\sin(\theta_2+\frac{\pi}{12})d\theta_2.$$

The integral may be written

$$\int_0^{\frac{1}{6}\pi}\sec^2(\theta_2-\frac{\pi}{12})\left[\sin(\theta_2-\frac{\pi}{12})\cos(\theta_2-\frac{\pi}{12})\sin\frac{\pi}{6}\right]d\theta_2$$

=(after reductions similar to those above)

$$\tfrac{1}{2}\int_0^{\frac{1}{6}\pi}\frac{d\theta_2}{\cos(\theta_2-\frac{\pi}{12})}=\tfrac{1}{2}\log_e\left(\frac{1+\tan\frac{\pi}{24}}{1-\tan\frac{\pi}{24}}\right)$$

$\therefore$ The numerator reduces to

$$\left[\log_e\left(\frac{1+\tan\frac{\pi}{24}}{1-\tan\frac{\pi}{24}}\right)\right]^2.$$

It may also be shown that the denominator reduces to

$$\left[\log_e\left(\frac{1+\tan\frac{\pi}{24}}{1-\tan\frac{\pi}{24}}\right)\right]^2.$$

$\therefore \Delta=\frac{1}{8}a^2$.

And the mean area of dodecagon=area of 12 such triangles $=\frac{3}{2}a^2$.

Solutions of this problem were received from G. B. M. Zerr and the Proposer, the latter furnishing two solutions.